TECHNICAL UNIVERSITY OF KOŠICE
Faculty of Mechanical Engineering

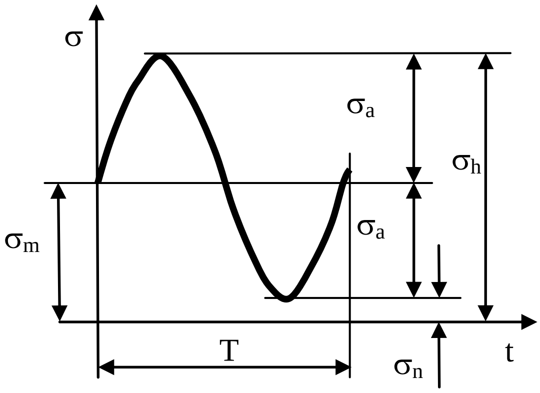
DESIGN of MACHINES and MACHINES PART

Part no: **13** Lecturer: prof. Ing. Robert Grega, PhD.

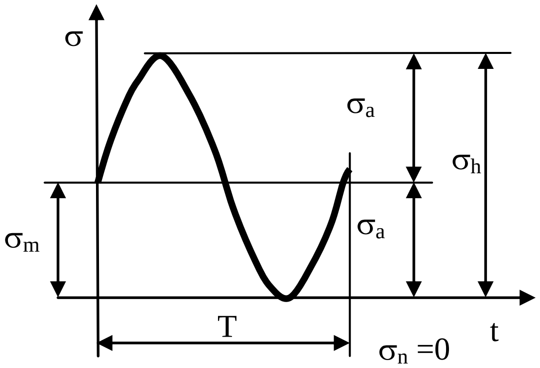
Fatigue Strength - Safety factor

Characterizing Fluctuating Stresses

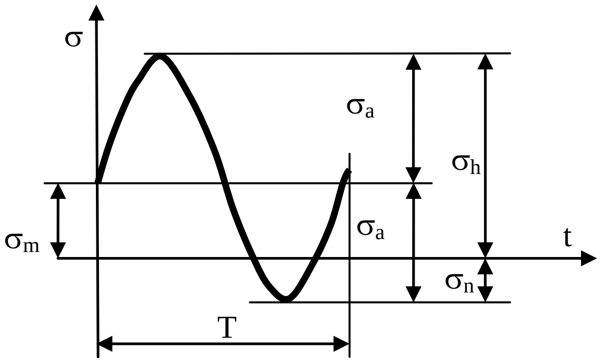
Sinusoidal fluctuating stress



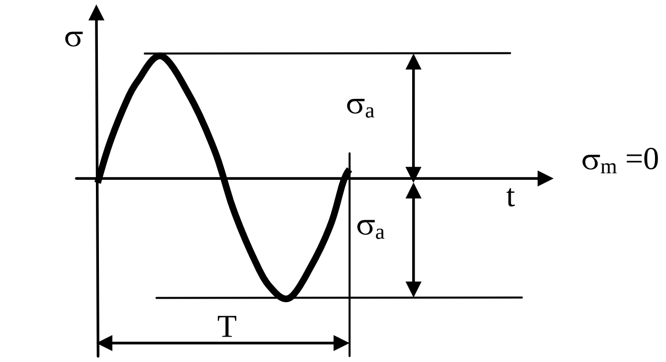
Repeated stress



Completly reversed Sinusoidal fluctuating stress



Symetrical Completly reversed Sinusoidal fluctuating stress



maximum stress:

$$\sigma_h=\sigma_m+\sigma_a$$

minimum stress:

$$\sigma_n=\sigma_m-\sigma_a$$

midrange component:

$$\sigma_m=\frac{\sigma_h+\sigma_n}{2}=\frac{\sigma_{max}+\sigma_{min}}{2}$$

amplitude component:

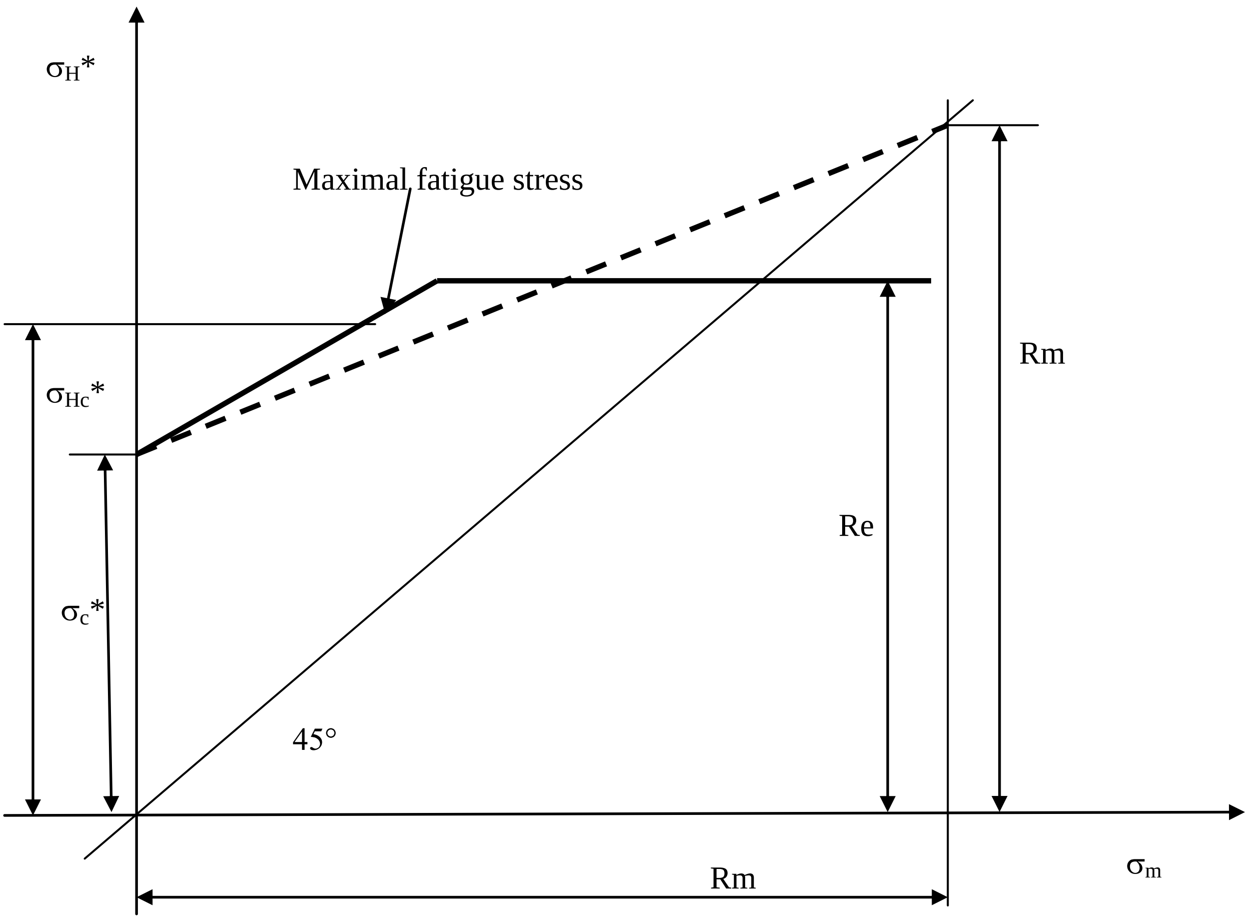
$$\sigma_a=\frac{\sigma_h-\sigma_n}{2}=\frac{\sigma_{max}-\sigma_{min}}{2}$$

Stress ratio:

$$r=\frac{\sigma_n}{\sigma_h}$$

The basic design of Smith diagram

Definition of Fatigue Strength limit by different References:
Reference: L.Málik a kol. , Konštruovanie II, Edis v Žiline 2013



σ_c* - fatigue stress of Symetrical Completly reversed Sinusoidal fluctuating stress
σ_{Hc}* - fatigue stress of Repeated stress
σ_m – midrange stress
Applies to a smooth test specimen:

$$\sigma_H = \sigma_c + (1 - \psi\ ).\sigma_m \quad a \quad R_e = \sigma_H$$

Applies to the notched part:

$$\sigma_H{}^* = \sigma_c{}^* + (1 - \psi^*).\sigma_m \quad a \quad \sigma_H{}^* = \sigma_k{}^* = R_e$$

If ψ is the coefficient of notch sensitivity of the material to cycle asymmetry, then:

$$\psi^* = \frac{\psi}{\beta}$$

β – notch factor
With classical methods of fatigue strength, it is necessary to know the effect of notch on fatigue strength. Due to the complexity of the problem, it is solved by using the notch coefficient.

The notch factor is defined as the ratio of the fatigue limit of the smooth sample to the fatigue limit of the notched sample.

$$\beta = \frac{\sigma_c}{\sigma_{cn}}$$

The increased stress is concentrated mainly in the root of the notch. The effect of the notch can be defined using the notch design factor α.

$$\alpha = \frac{\sigma_{max}}{\sigma_n}$$

σ_{max} a σ_n – the stresses are maximum and minimum calculated according to the classical methods of elasticity and strength

The following factors also influence the fatigue limit:

- part size - part size factor: ν
- surface quality - surface quality factor: ε_p

After taking into account the above factors, the fatigue limit for the real component will be:

For fluctuating bending stress:

$$\sigma_c{}^* = \frac{\sigma_c . \nu_\sigma . \varepsilon_p}{\beta_\sigma}$$

For fluctuating torsion stress:

$$\tau_c{}^* = \frac{\tau_c . \nu_\tau . \varepsilon_p}{\beta_\tau}$$

For ductile materials it is possible to use the relation:

$$\tau_c = 0{,}57 . \sigma_c$$

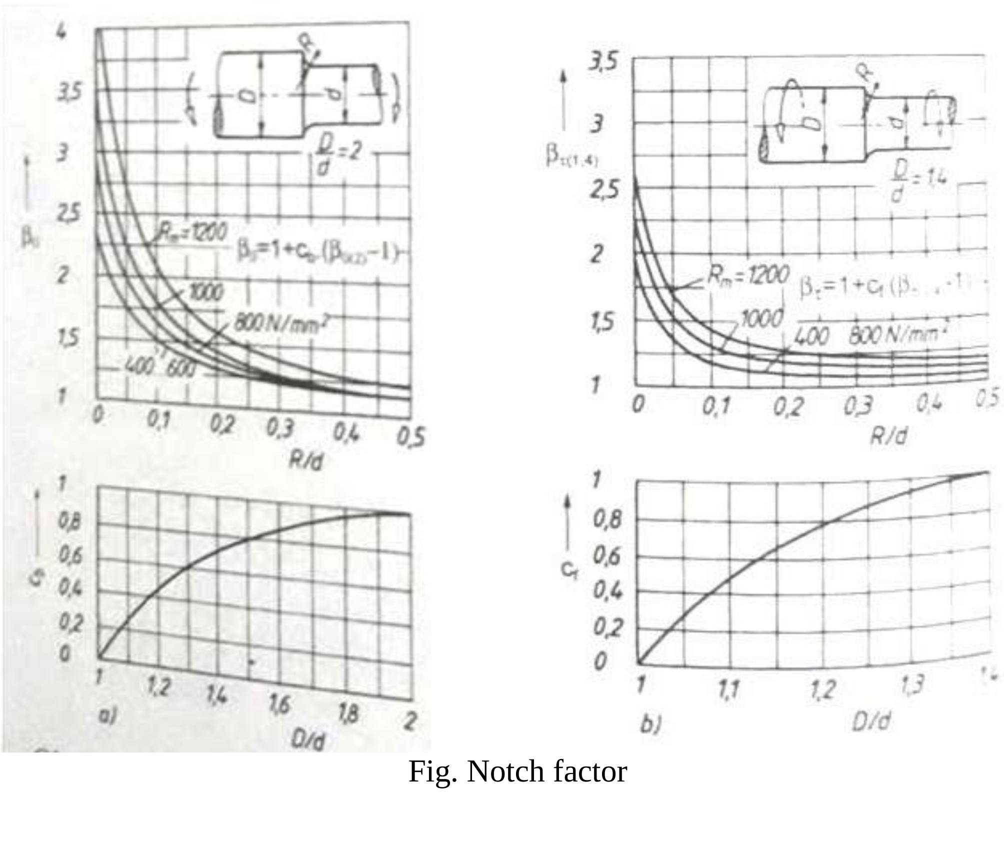


Fig. Notch factor

For tension, bending, compressed stress:

$$\psi_o = \frac{(2.\sigma_c - \sigma_{HC})}{\sigma_{HC}}$$

$$\psi_o{}^* = \frac{(2.\sigma_c{}^* - \sigma_{HC}{}^*)}{\sigma_{HC}{}^*}$$

For torsion and shear stress:

$$\psi_k = \frac{(2.\tau_c - \tau_{HC})}{\tau_{HC}}$$

$$\psi_k{}^* = \frac{(2.\tau_c{}^* - \tau_{HC}{}^*)}{\tau_{HC}{}^*}$$

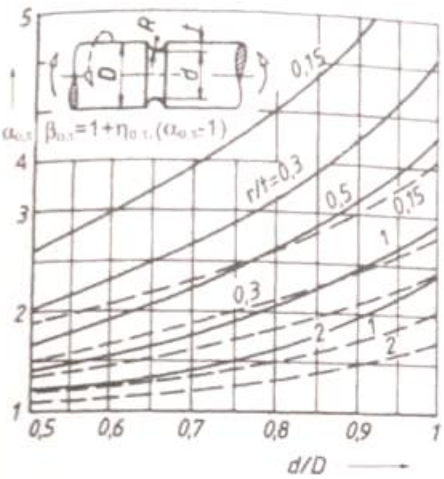
For repeated tension and bending stress:

$$\sigma_{HC} = \frac{2.\sigma_c}{1+\psi} = (1{,}6 - 2).\sigma_C$$

$$\sigma_{HC}{}^* = \frac{2.\sigma_c{}^*}{1+\psi^*} = \left(\frac{2.\beta}{\beta+\psi}\right).\sigma_C{}^*$$

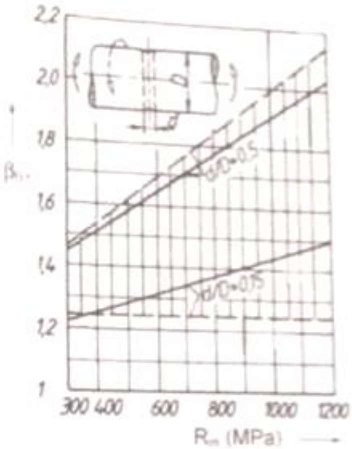
For torsion and shear stress:

$$\tau_{HC} = \frac{2.\tau_c}{1+\psi_k} = (1{,}74 - 2).\tau_C$$



Bending strength
Torsion strength

Fig. notch design factor



Bending strength
Torsion strength

Fig. Notch factor

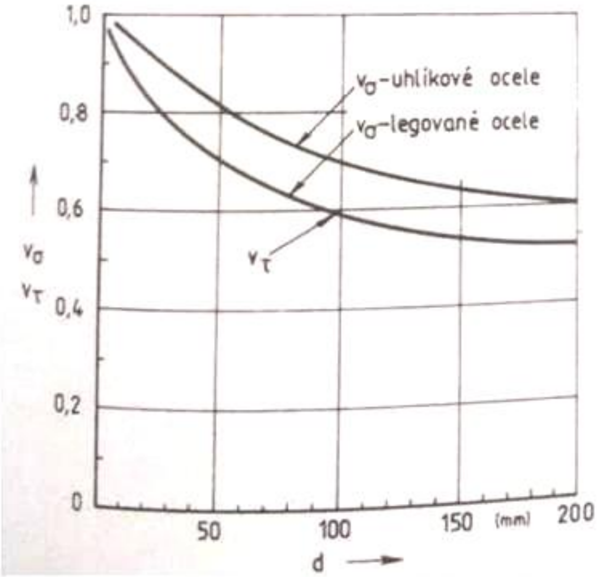
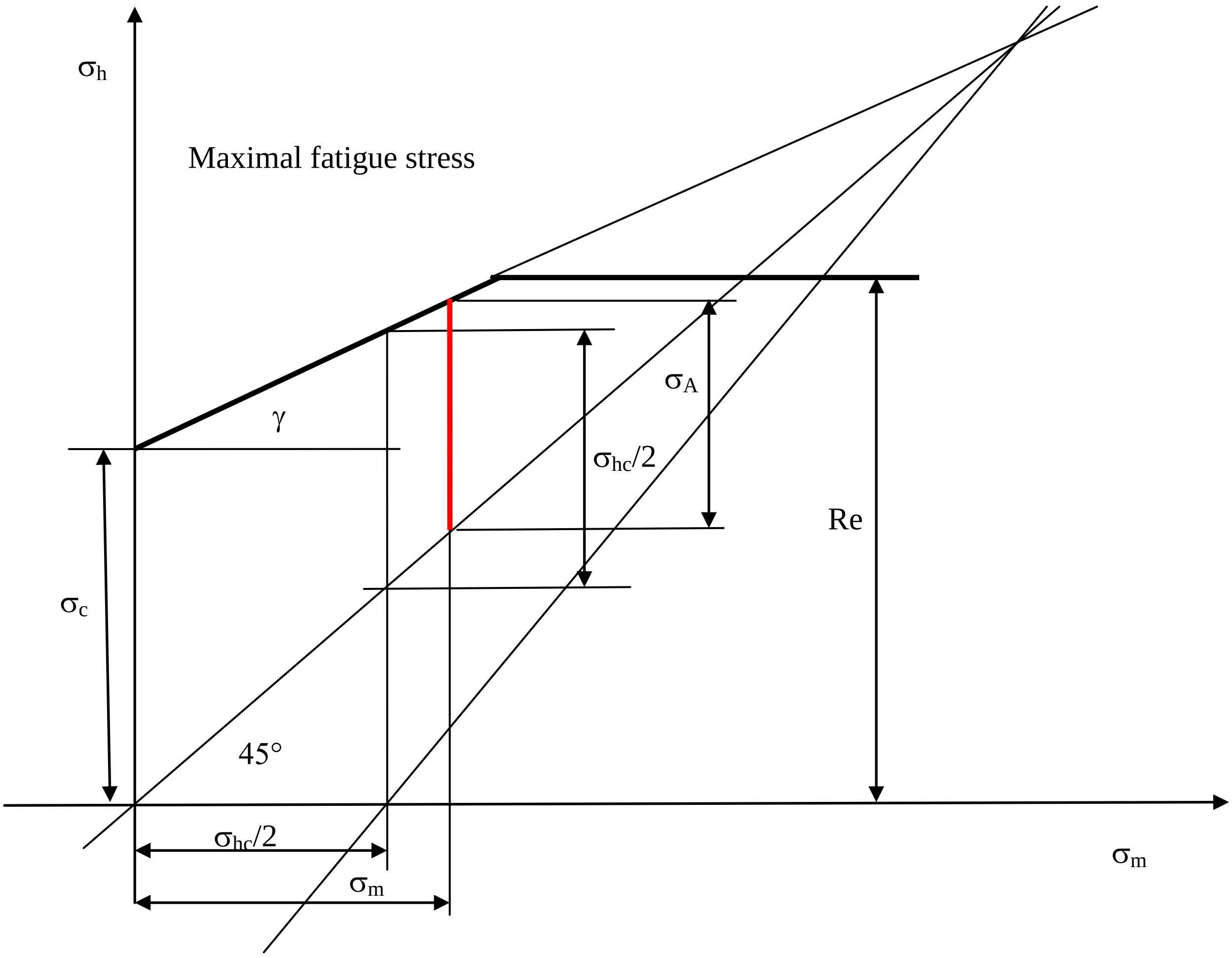


Fig. size factor

If the value of stress concentration **α** and material strength σ_{pt} is known, it is possible to determine the notch sensitivity coefficient **q** for which:

$$q = \frac{(\beta - \mathbf{1})}{(\alpha - \mathbf{1})} \leq 1$$

References:
[1] F. Boháček a kol. , Části a mechanizmy stroju I, Edičný stredisku VUT Brno 1984
[2] Richard G. Budynas, J. Keith Nisbett.: Shigley's Mechanical Engineering Design, Tenth Edition, Published by McGraw-Hill Education, 2 Penn Plaza, New York, NY 10121., 2015

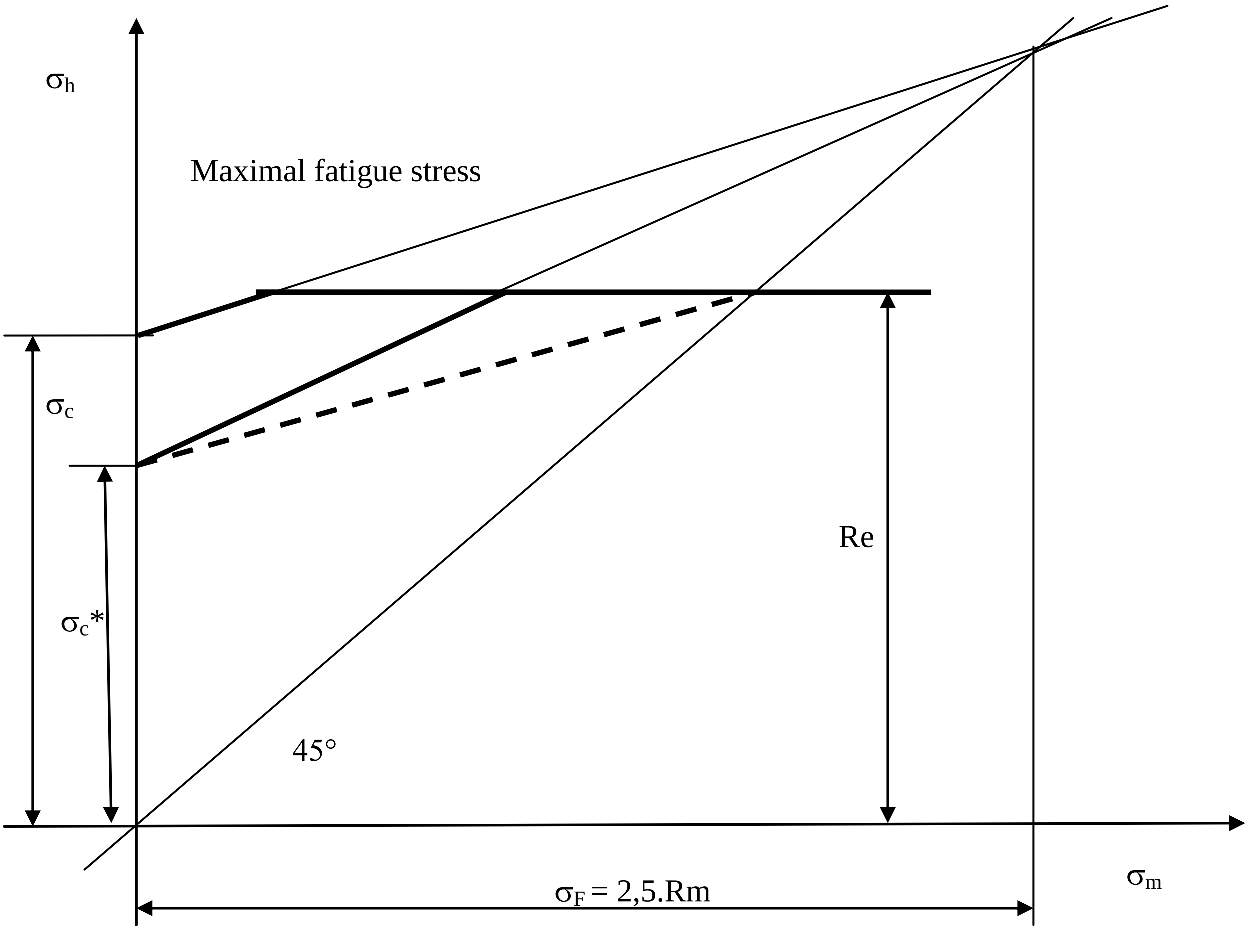


σ_{hc} – Maximal fatigue stress
σ_c – fatigual stress of symmetrical fluctuating stress
σ_m – midrange stress
σ_A– amplitude stress

$$\sigma_A = \sigma_{hc} - \sigma_m = \sigma_c - \psi.\sigma_m$$

$$\gamma = \tan^{-1}(1-\psi)$$

ψ – the factor depended on the strength of the material
The use of this diagram design is for the area of permanent strength, where no fatigue crack propagation occurs. For the area of time fatigue strength, the upper fatigue limit is adjusted according to the slope of the service life curves, see. recommended literature.



σ_c* - fatigue limit for notched part
σ_c – fatigue limit for component without notch
σ_m – midrange stress
σ_F – fictitious stress - different according to the type of material

If this design is used for brittle materials, it needs to be replaced Re with Rm.

Safety factor

The safety factor **kσ** determines how many times the operating stress must increase until it reaches the limit state. In practice, we usually determine two types of safety, namely safety amplitude and safety to the upper limit of fatigue

Amplitude safety factor: recommended value k_a=2,5-4

$$k_a = \frac{\sigma_A}{\sigma_a}$$

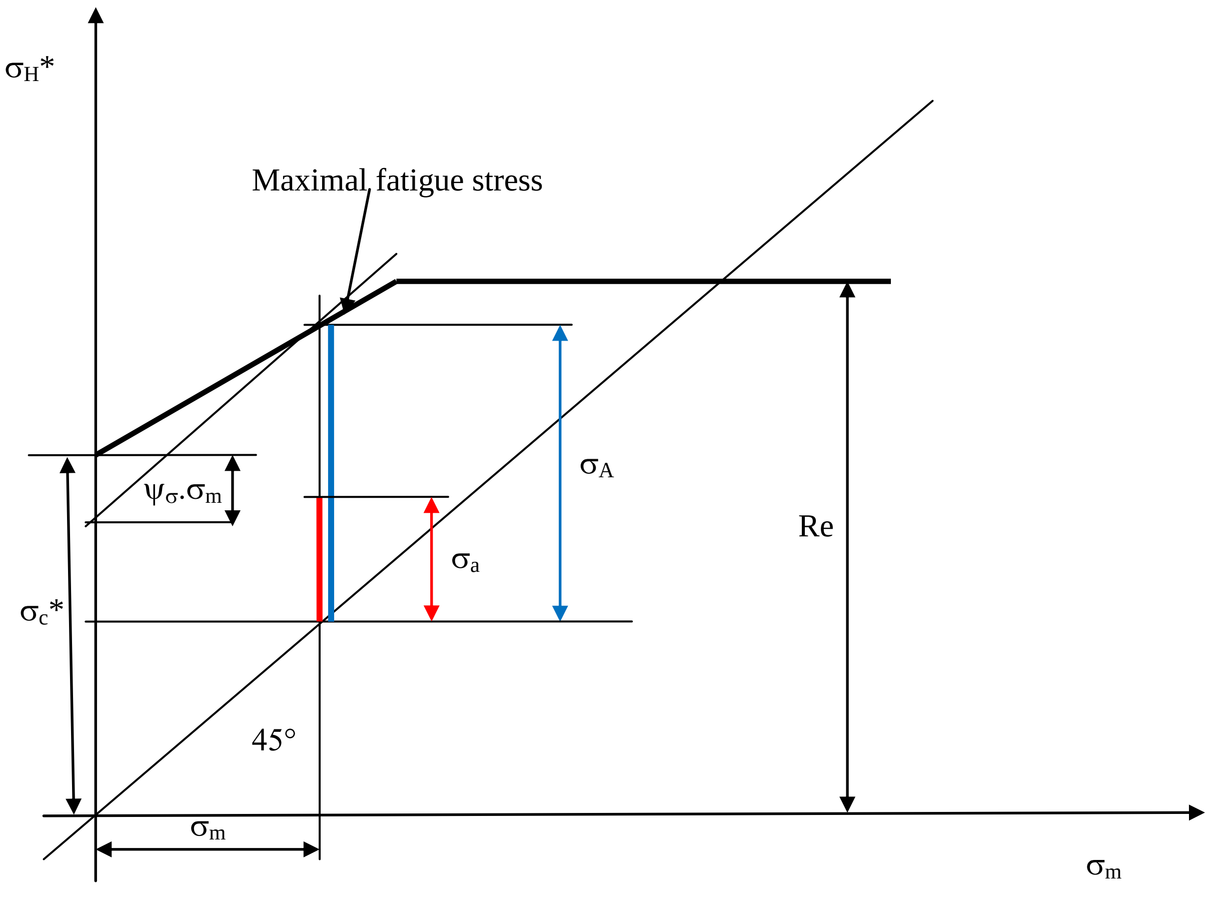
Safety factor for Maximal fatigue limit: the recommended value k=1,25-2,5

$$k = \frac{\sigma_H}{\sigma_h}$$

Usually for σ_H is substitution R_e

In practice, there are three typical cases of changes in operating stresss:

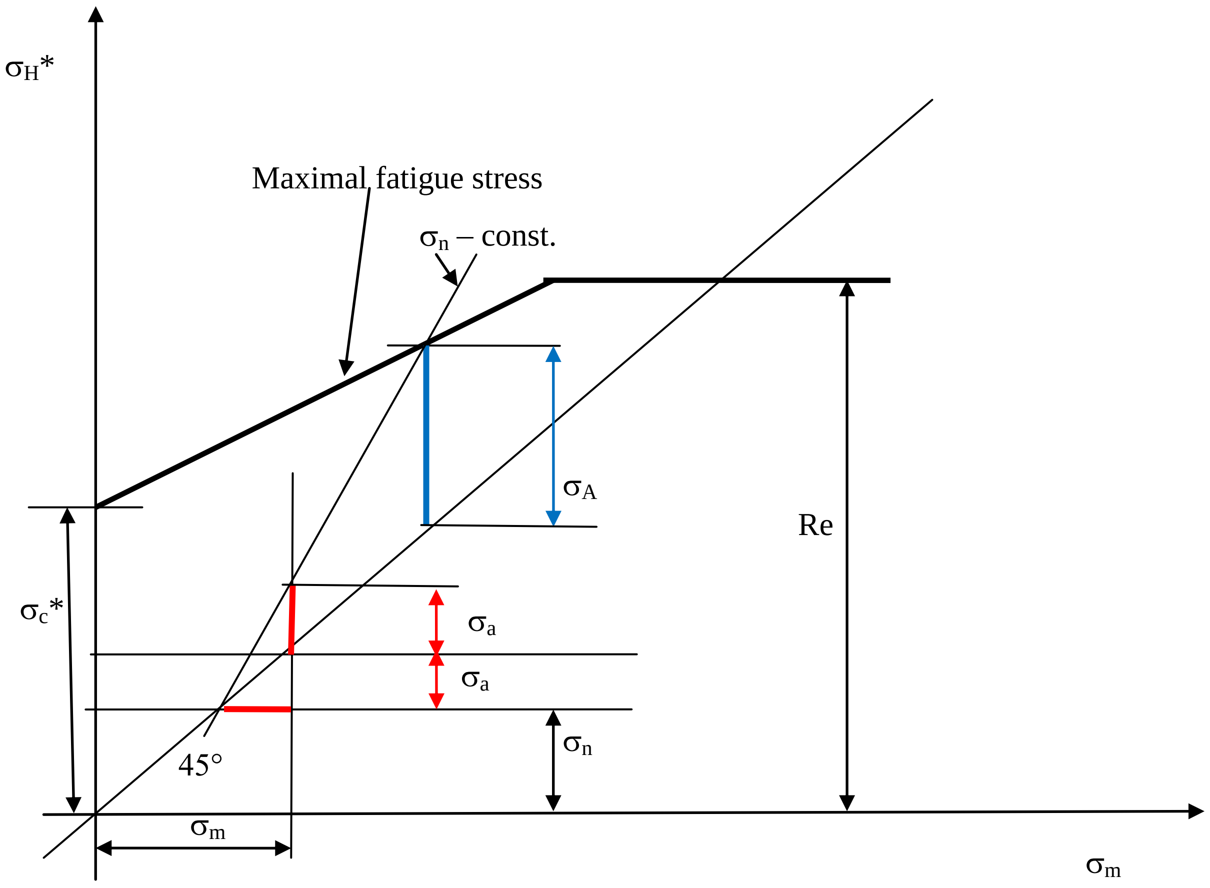
a) The midrange stress σ_m is constant and the amplitudes stress change σ_a.



Safety factor:

$$k_\sigma = \frac{\sigma_A}{\sigma_a} = \frac{\sigma_c{}^* - \psi_\sigma . \sigma_m}{\sigma_a}$$

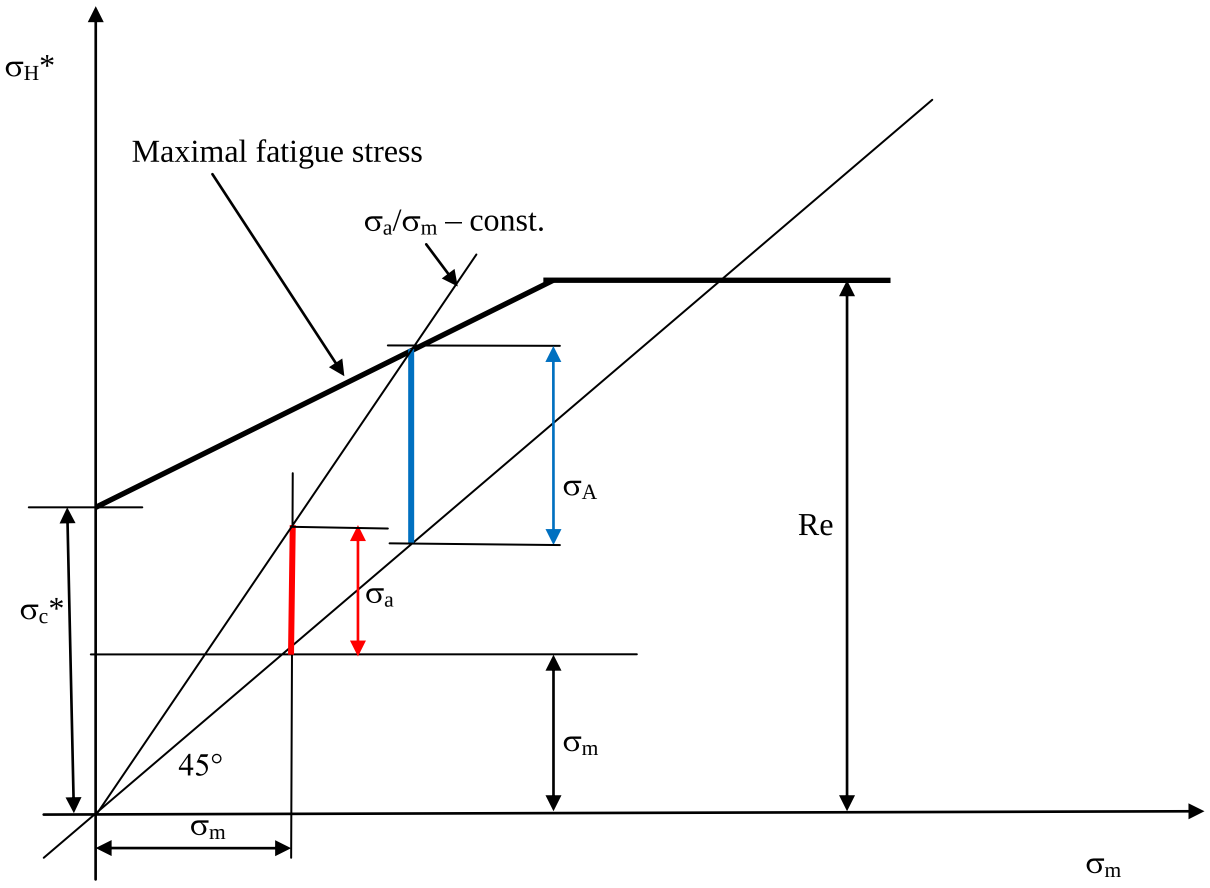
b) The minimal stress σ_n is constant
This case is a typical case for a preloaded screw.



Safety factor:

$$k_\sigma = \frac{\sigma_A}{\sigma_a} = \frac{\sigma_c{}^* - \psi_\sigma.(\sigma_m - \sigma_a)}{\sigma_a + \psi_\sigma.\sigma_a} = \frac{\sigma_c{}^* - \psi_\sigma.\sigma_n}{(1+\psi_\sigma).\sigma_a}$$

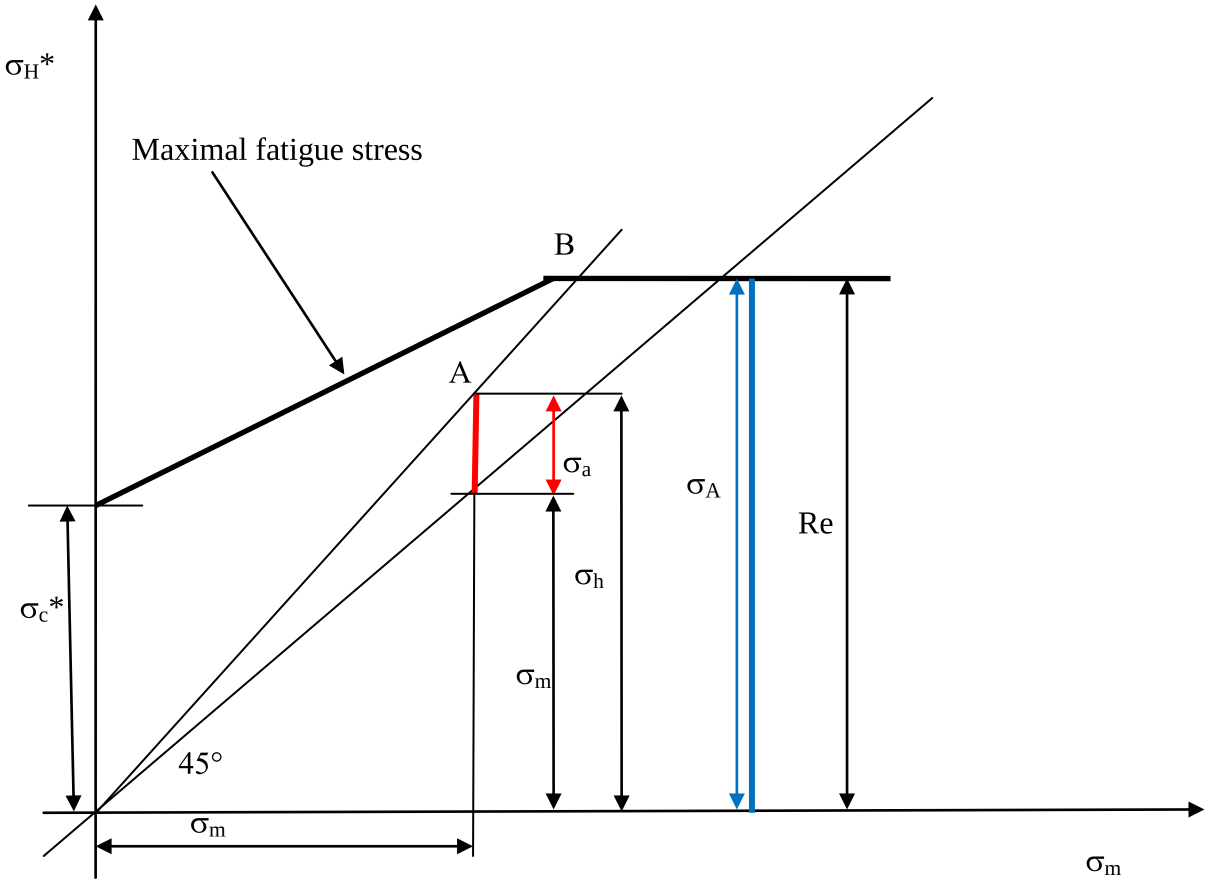
c) Ratio of stress σ_a / σ_m or σ_h / σ_m is constatnt



Safety factor:

$$k_\sigma = \frac{\sigma_A}{\sigma_a} = \frac{\sigma_c{}^*}{\sigma_a + \psi_\sigma . \sigma_m}$$

d) in case the line AB intersects the limit Re, it is necessary to determine the safety to the limit of plastic deformations



Safety factor:

$$k_\sigma = \frac{R_e}{\sigma_h} = \frac{R_e}{\sigma_a + \sigma_m}$$

In the case of combined bending and torsional stresses (normal and shear stresses), the relations:

Theory HMH:

$$\sigma_{red} = \sqrt{\sigma_a^2 + 3.\tau_a^2}$$

Theory τ_{max}:

$$\sigma_{red} = \sqrt{\sigma_a^2 + 4.\tau_a^2}$$

Similarly, according to the above procedures, this is also the case when determining the degree of safety in torsion $\mathbf{k_t}$.

Safety factor for fluctuating bending and torsion are:

$$k = \frac{\sigma_c}{\sigma_r}$$

Safety factor for bending:

$$k_o = \frac{\sigma_c}{\sigma_a}$$

Safety factor for torsion:

$$k_k = \frac{\tau_c}{\tau_a}$$

Finally safety factor:

$$k = \frac{k_o.k_k}{\sqrt{(k_o^2 + k_k^2)}}$$