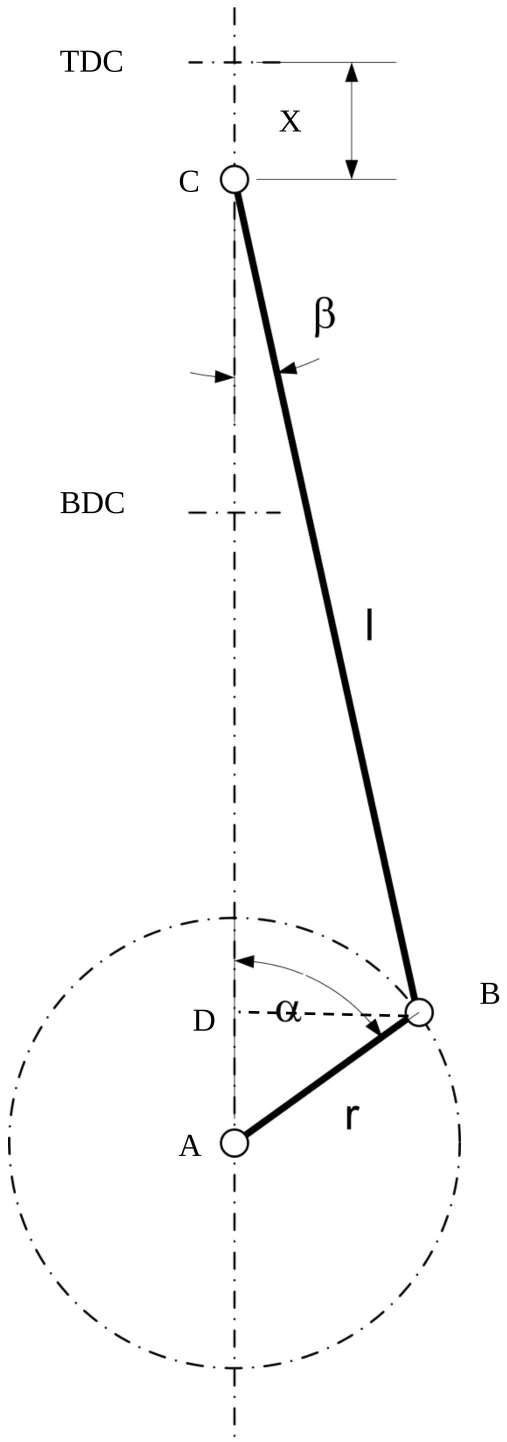
Kinematics of crank mechanism

A crankshaft is the mechanical part able to perform conversion between rotating and reciprocating motion.



Calculation:
x - distance of piston position
r- radius of crankshaft
l - length of connecting rod
α - angel of rotation crankshaft

λ=r/l

Then:
Displacement - x:

$$x = (l + r) - r.cos\alpha - l.cos\beta$$

$$x = r.[1 - cos\alpha + \frac{1}{\lambda}(1 - cos\beta)]$$

$$cos\beta = \sqrt{1 - sin^2\beta}$$

$$from\ \Delta ADB\ and\ \Delta BDC \rightarrow l.\sin\beta = r.\sin\alpha \rightarrow \sin\beta = \lambda.sin\alpha$$

$$\beta = arsin(\lambda.sin\alpha)$$

$$cos\beta = \sqrt{1 - \lambda^2.sin^2\alpha}$$

$$x = r.[1 - cos\alpha + \frac{1}{\lambda}\left(1 - \sqrt{1 - \lambda^2.sin^2\alpha}\right)]$$

$$x \cong r.[1 - cos\alpha + \frac{1}{2}\lambda.sin^2\alpha]$$

Piston speed (velocity) - c:
Average piston speed: $c_s = 2z.n$
z - stroke
n- revolution of crankshaft
Current Piston speed:

$$\omega = d\alpha/dt$$

$$c = \frac{dx}{dt} \Longrightarrow c = r.\omega.sin\alpha[1 + \frac{\lambda.cos\alpha}{\sqrt{1 - \lambda^2.sin^2\alpha}}]$$

Simply:

$$c \cong r.\omega.(sin\alpha + \lambda.sin\alpha.cos\alpha)$$

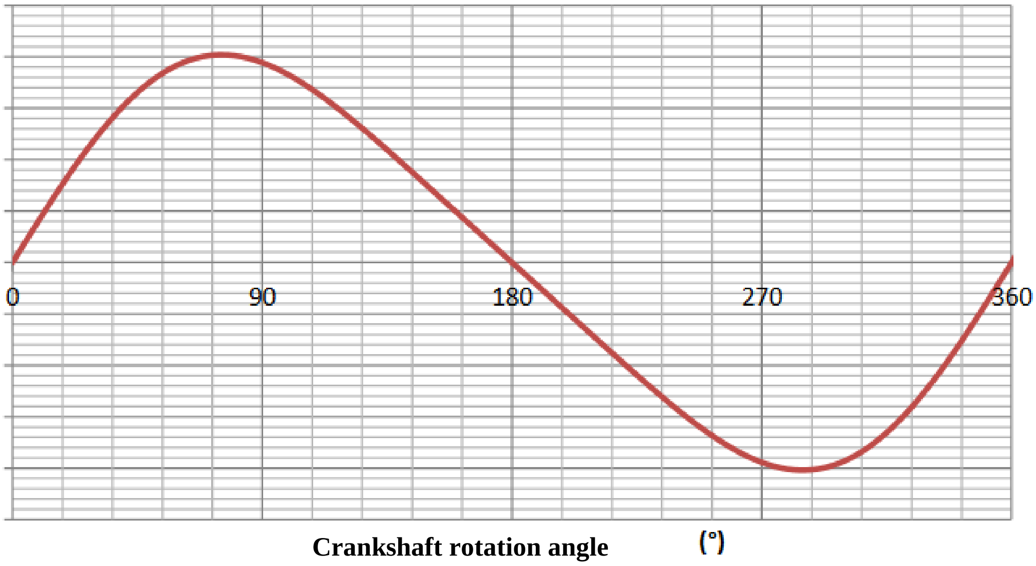
or

$$c \cong r.\omega.(sin\alpha + \frac{\lambda}{2}.sin2\alpha)$$

Maximal speed of piston:

$$c = r.\omega.\sqrt{1+\lambda^2}$$

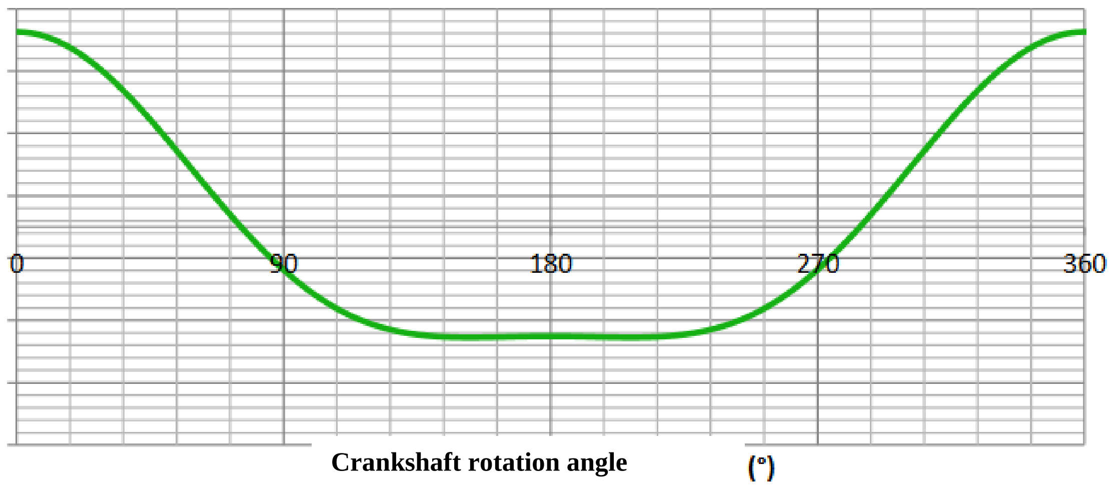
Speed graph:



Acceleration - a:
if ω=const.

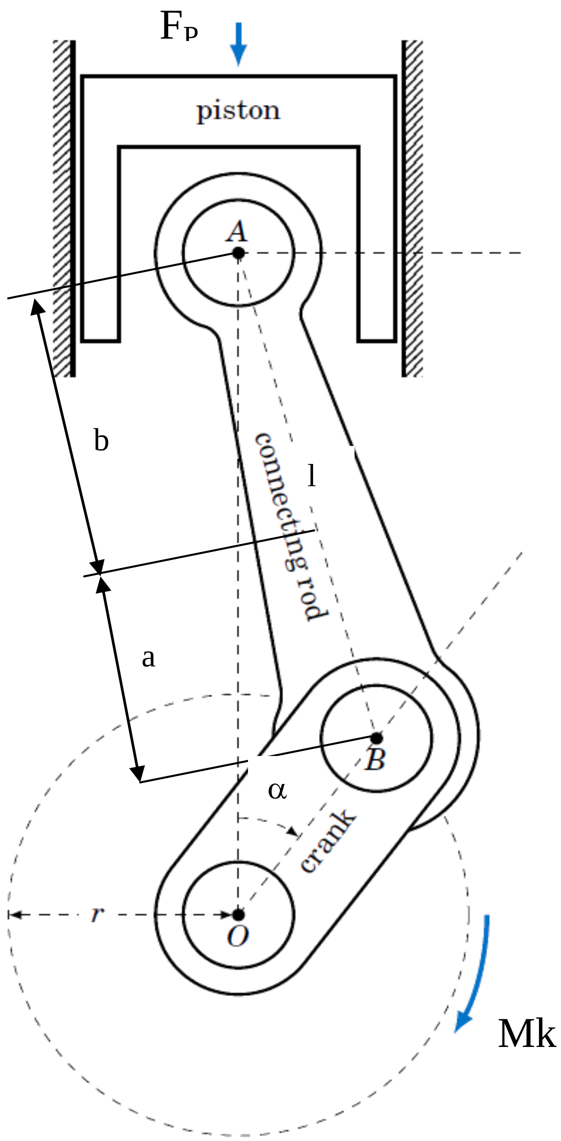
$$a = \frac{dx^2}{dt^2} \Longrightarrow a \simeq r.\omega^2.(cos\alpha + \lambda.cos2\alpha)$$

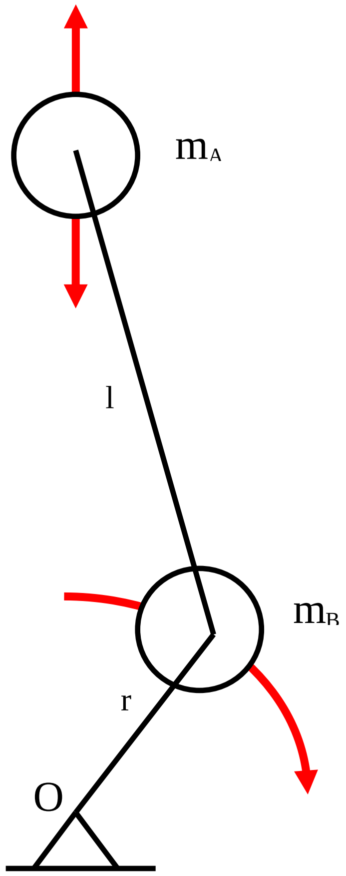
Graph of acceleration:



Dynamics of crank mechanism

1. Substitution of mass





Each part of the crank mechanism will have a weight (mass).
Mass - m_A .
In point "A" we will concentration mass of reciprocating motion parts (piston, piston rings, wrist pin, segment of connecting rod)
Mass - m_B
In point "B" we will concentration mass of rotating motion parts (segment of crankshaft, crankpin bearing journal, segment of connecting rod)
Reduction of mass of connection rod:

$$m_o = m_{op} + m_{or}$$

m_o- total mass of conection rod

m_{op} - segment of connecting rod reduced to the motion mass
m_{or} - segment of connecting rod reduced to rotating mass

$$m_{op} .b= m_{or}. a$$
$$m_{or} = m_o.b/l$$

2. Power force

$$F_p=S.p(\alpha)$$

3. Calculation of inertia force

Inertia force of rotating parts:

$$F_{zr}=m_B.r.\omega^2$$
$$F_{zr}=(m_{or} + m_c + m_{khred}.x_r/r).r.\omega^2$$

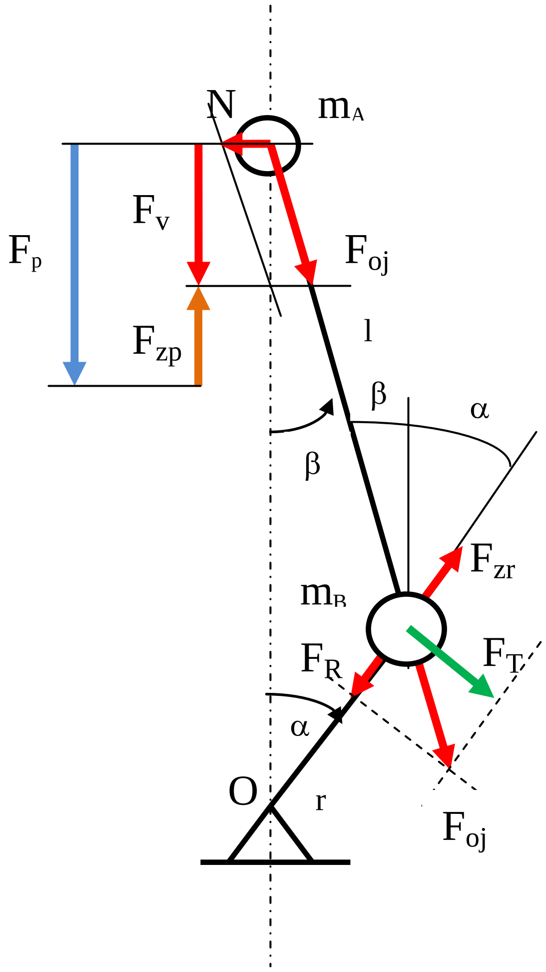
Inertia force of motion parts:

$$F_{zp}=m_A.a$$
$$F_{zp}=(m_{op} + m_p + m_{pr} +m_{pp}).a$$

$$F_{zp} = m_A.r.\omega^2.(cos\alpha + \lambda.cos2\alpha)$$
$$F_{zp} = m_A.r.\omega^2.cos\alpha + m_A.r.\omega^2.\lambda.cos2\alpha$$
$$F_{zp} = F_{zpI} + F_{zpII}$$

Inertia force acting against opposite force.
F_{zpI} - Inertia force of 1st order.
F_{zpII} -Inertia force of 2nd order.

4. Force in crank mechanism



Finally force on piston (wrist) pin:

$$\overline{F_v} = \overline{F_p} + \overline{F_{zp}}$$

Normal force - eliminated on side of cylinder:

$$N = F_v . tg\beta$$

Force of connecting rod:

$$F_{oj} = \frac{F_v}{cos\beta}$$

Divided the force of connecting rod on forces crankshaft pin:

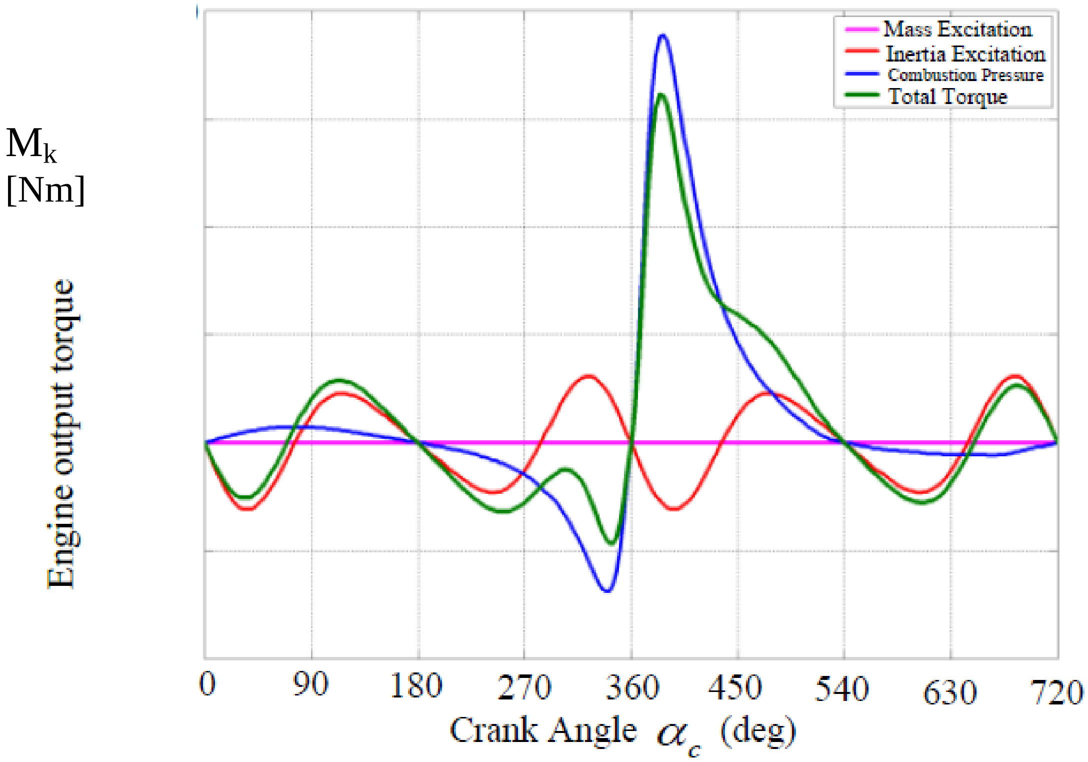
$$F_T = F_{oj} . \sin{(\alpha + \beta)}$$

$$F_R = F_{oj} . \cos{(\alpha + \beta)}$$

Torque of crankshaft:

$$M_k = F_T . r$$

Graph of torque by one cylinder engine:



Graph of torque by multi cylinder engine: